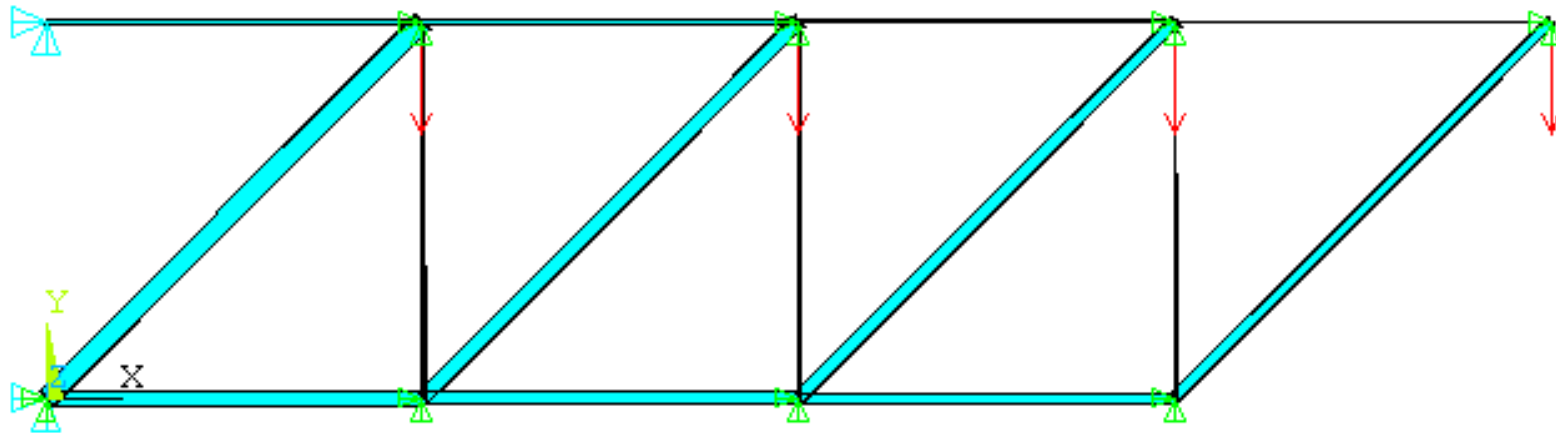
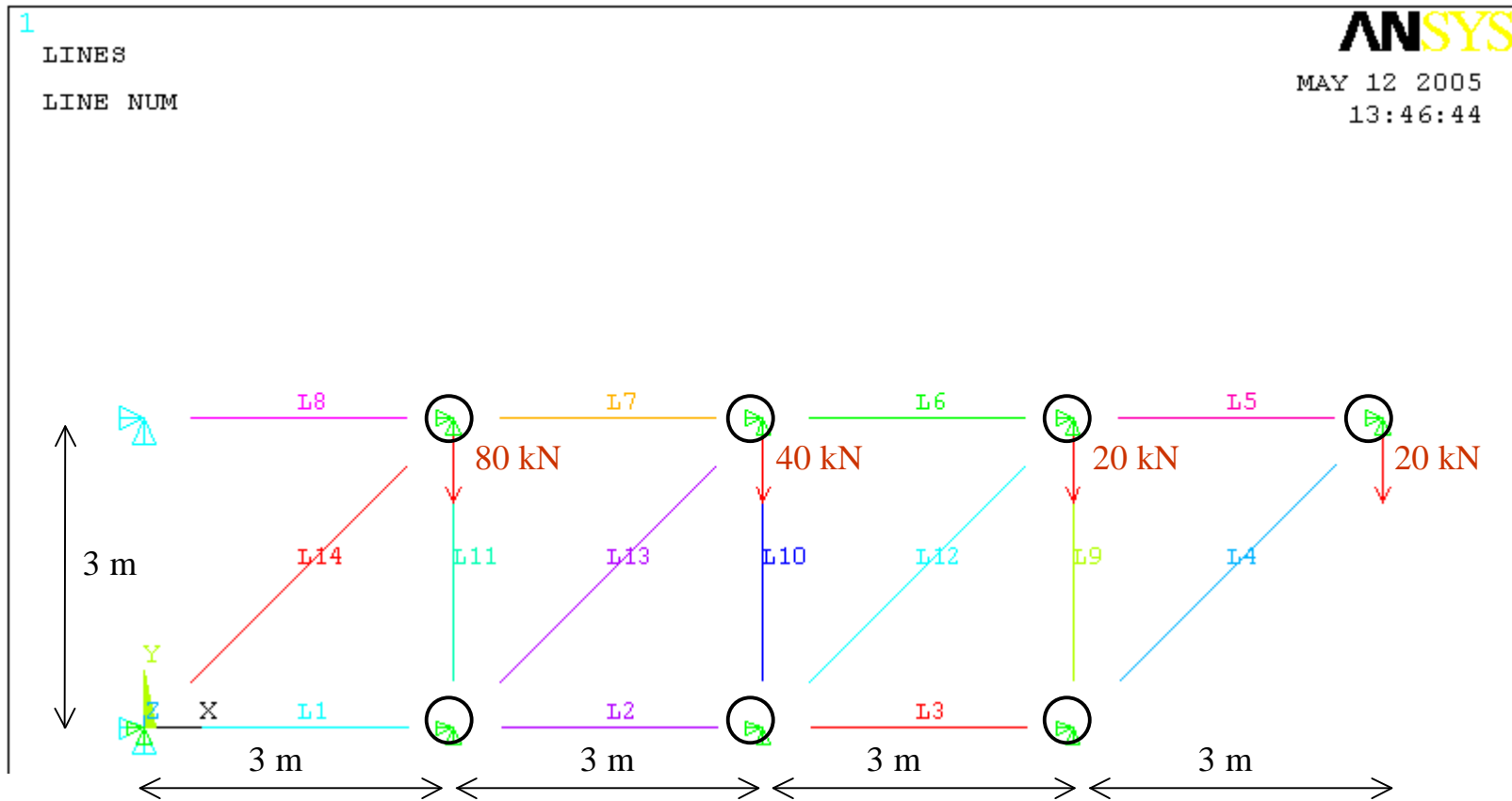


# A case study

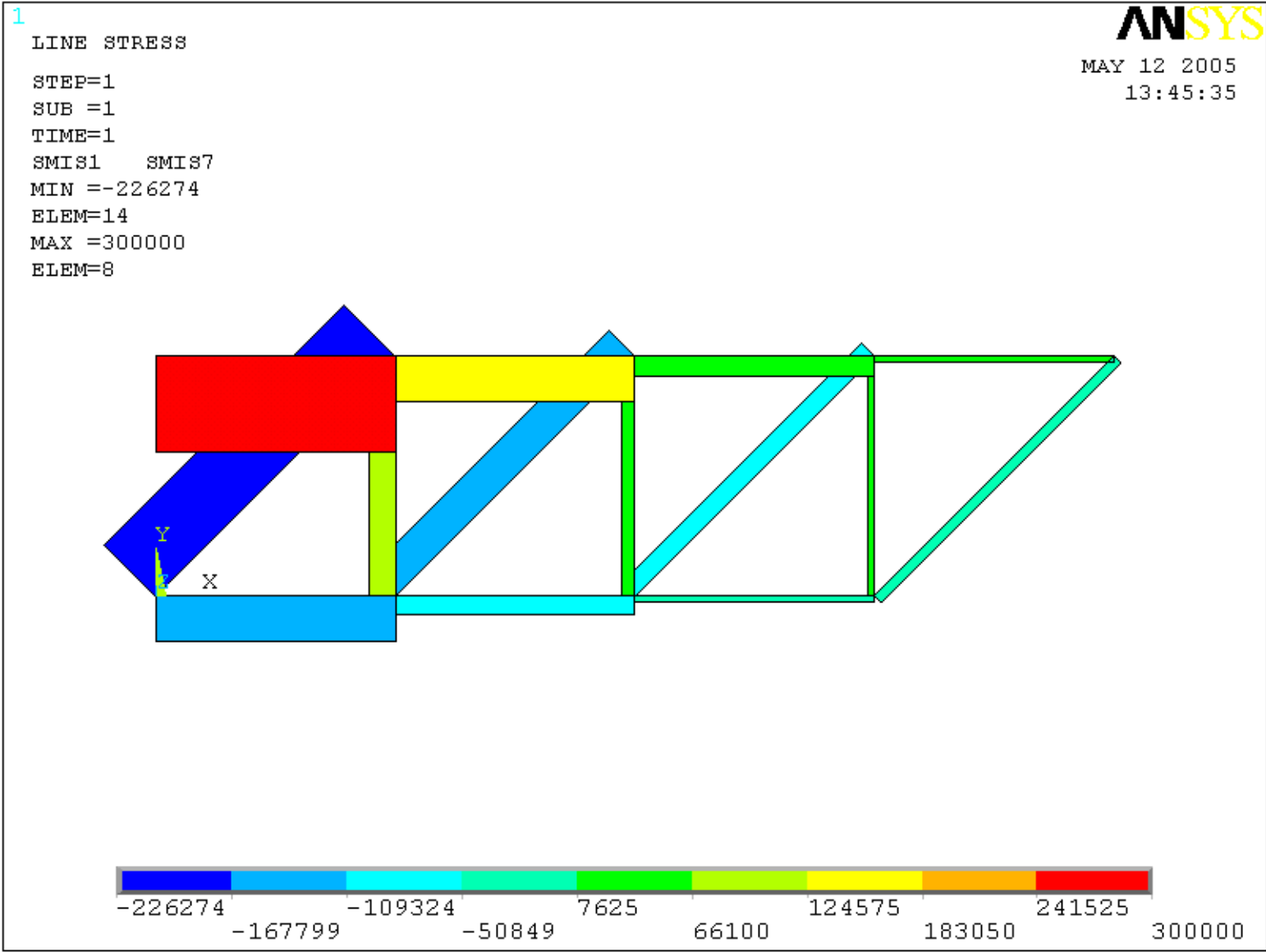
The case study example is taken from the book of  
Optimal Designs - Structures and Materials - Problems and Tools -  
Pauli Pedersen (2003), 33-35 p.



# Numbering of truss elements

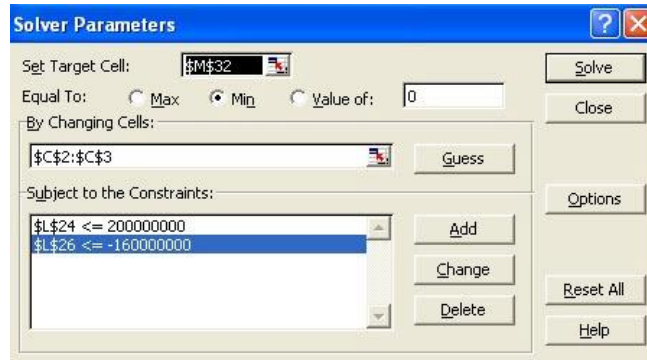


# An axial force distribution in truss structure

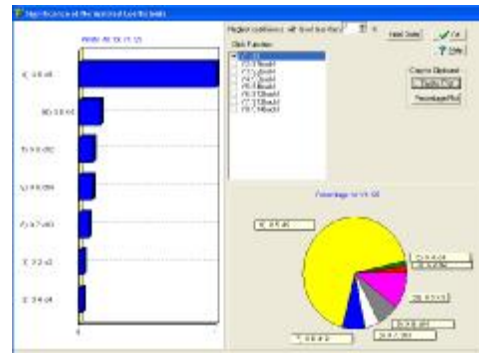
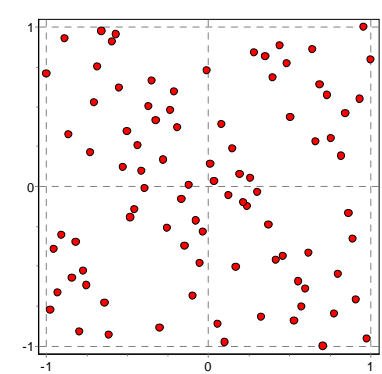


# Optimisation strategy

## 1) Direct optimisation using Excel optimisation solver



## 2) Indirect optimisation using Response Surface Method and Design of Experiments



The screenshot shows the Minitab software interface displaying a data table. The table has columns for Index, Min, Max, Mean, and others. The data is organized into a grid, and the 'Functional Constraints' section is visible at the bottom.

| Index   | Min  | Max      | Mean   | Stdev  | Lower  | Upper  |
|---------|------|----------|--------|--------|--------|--------|
| 3157-01 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-02 | 0.08 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-03 | 0.06 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-04 | 0.07 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-05 | 0.08 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-06 | 0.08 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-07 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-08 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-09 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-10 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-11 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-12 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-13 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-14 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-15 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-16 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-17 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-18 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-19 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-20 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-21 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-22 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-23 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-24 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-25 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-26 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-27 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-28 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-29 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-30 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-31 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-32 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-33 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-34 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-35 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-36 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-37 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-38 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-39 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-40 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-41 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-42 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-43 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-44 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-45 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-46 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-47 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-48 | 0.10 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-49 | 0.11 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3157-50 | 0.14 | 0.001011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

# Estimation of buckling criteria by reduction coefficient as function of columns slenderness

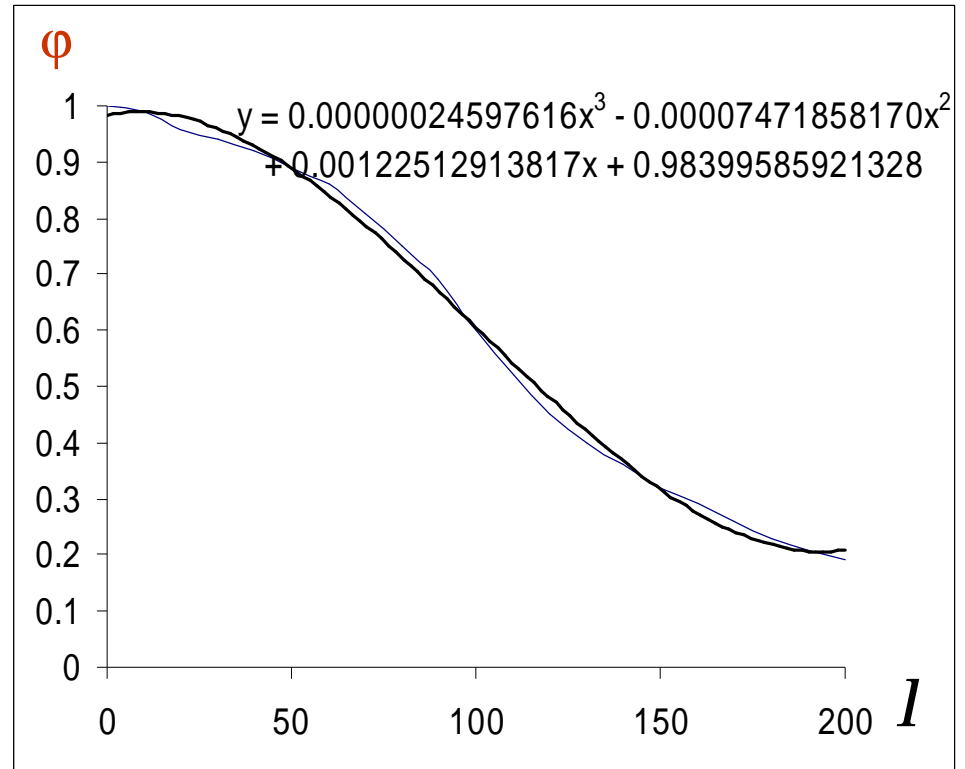
$$s \geq \frac{P}{A} \quad \text{Strength criteria}$$

$$s \geq \frac{P}{jA} \quad \text{Buckling criteria}$$

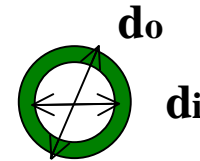
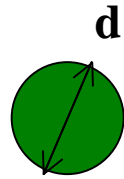
$$l_{red} = l \times m$$

$$I = \frac{l_{red}}{i}$$

$$i = \sqrt{\frac{I}{A}}$$



# Optimum cross-section estimation by EXCEL



d – diameter of the solid section bar

A – cross-section area

I – cross-section moment of inertia

do – outer diameter of the hollow section bar

di – inner diameter of the hollow section bar

t – wall thickness of the hollow section bar

$\lambda$  – slenderness of the hollow section bar

i – radius of inertia of the hollow section bar

$\phi$  – buckling reduction coefficient

| Element number | d<br>m | A<br>m <sup>2</sup> | I<br>m <sup>4</sup> |
|----------------|--------|---------------------|---------------------|
| <b>d5;d9</b>   | 0.011  | 0.00010             | 7.96E-10            |
| <b>d6</b>      | 0.020  | 0.00030             | 7.16E-09            |
| <b>d7</b>      | 0.030  | 0.00070             | 3.90E-08            |
| <b>d8</b>      | 0.044  | 0.00150             | 1.79E-07            |
| <b>d10</b>     | 0.016  | 0.00020             | 3.18E-09            |
| <b>d11</b>     | 0.023  | 0.00040             | 1.27E-08            |

| Element number | d0<br>m | A<br>m <sup>2</sup> | I<br>m <sup>4</sup> | di<br>m | m<br>% | t<br>mm |
|----------------|---------|---------------------|---------------------|---------|--------|---------|
| <b>d3;d4</b>   | 0.083   | 0.00053             | 4.41E-07            | 0.079   | 0.95   | 2.1     |
| <b>d2;d12</b>  | 0.102   | 0.00079             | 9.73E-07            | 0.097   | 0.95   | 2.5     |
| <b>d1</b>      | 0.121   | 0.00112             | 1.96E-06            | 0.115   | 0.95   | 3.0     |
| <b>d13</b>     | 0.123   | 0.00115             | 2.06E-06            | 0.117   | 0.95   | 3.1     |
| <b>d14</b>     | 0.158   | 0.00191             | 5.64E-06            | 0.150   | 0.95   | 3.9     |

| Element number | $\lambda$ | i<br>mm | $\phi$ |
|----------------|-----------|---------|--------|
| <b>s3;s4</b>   | 147.5     | 2.88    | 0.33   |
| <b>s2;s12</b>  | 121.0     | 3.51    | 0.47   |
| <b>s1</b>      | 71.8      | 4.18    | 0.78   |
| <b>s13</b>     | 100.3     | 4.23    | 0.60   |
| <b>s14</b>     | 78.0      | 5.44    | 0.74   |

# Optimum design using Excel solver

Design constraints – tensile stress 200MPa; compression stress –160 MPa

| Element number | N<br>kN | S<br>Pa   | S REAL<br>Pa |
|----------------|---------|-----------|--------------|
| T1             | 20000   | 2.00E+08  |              |
| T8             | 300000  | 2.00E+08  |              |
| C3;C4          | -28000  | -1.60E+08 | -5.25E+07    |
| C2;C12         | -60000  | -1.60E+08 | -7.58E+07    |
| C1             | -140000 | -1.60E+08 | -1.24E+08    |
| C13            | -111300 | -1.60E+08 | -9.66E+07    |
| C14            | -226300 | -1.60E+08 | -1.19E+08    |

N - Axial force in bar element

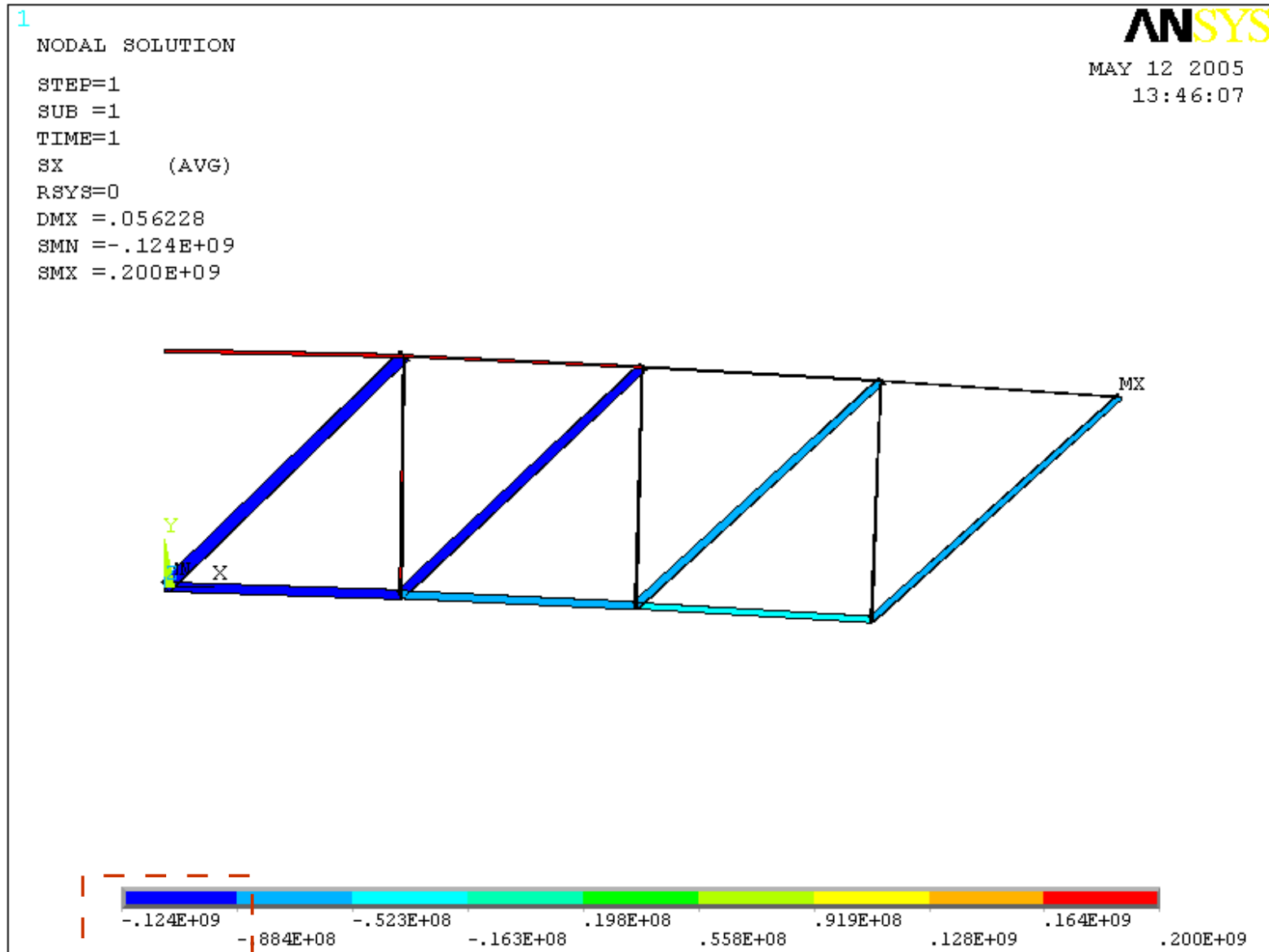
$\sigma$ – Stress in bar element with buckling reduction coefficient

$\sigma_{REAL}$  –Real stress in bar without buckling reduction coefficient

Cost function – minimisation of truss weight

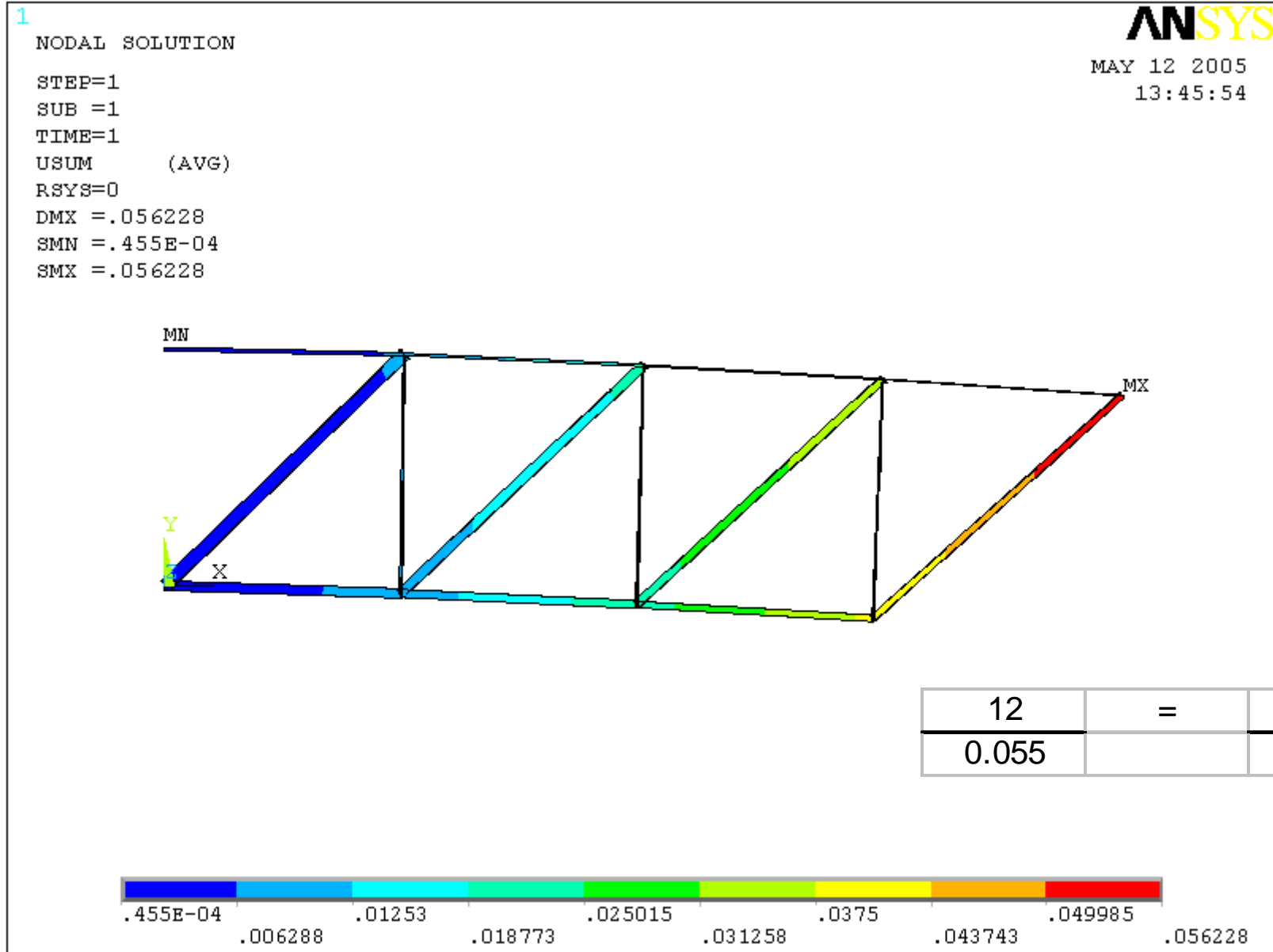
|               |       |                |
|---------------|-------|----------------|
| <b>Weight</b> | 714.4 | kg             |
| <b>Volume</b> | 0.095 | m <sup>3</sup> |

# Verification with FEM ANSYS normal stresses distribution



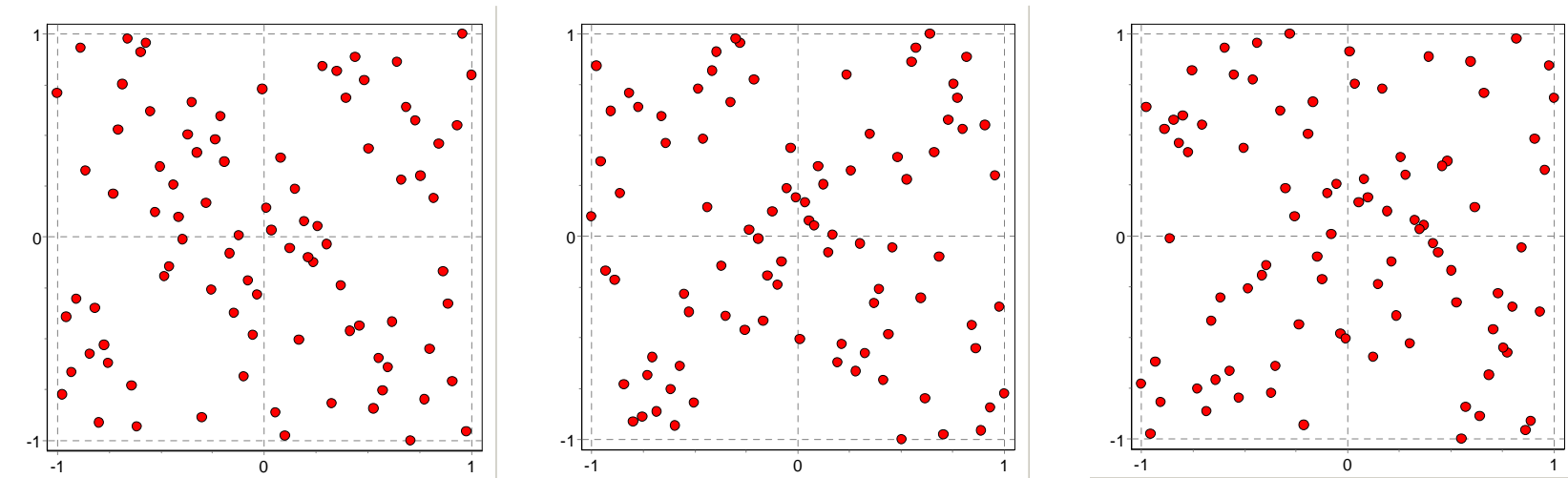


# Verification with FEM ANSYS deflection



# Design of Experiments

## Latin – Hypercube - plan of experiments



**K – number of design variables**

**2nd order:  $L=1+2K+K(K-1)/2$ ;  $K=8$ ;  $L=45$ ;**

**Sample points of ED - 2nd order:  $N\sim 2L$ ;  $N=90$ ;**

**1st order:  $L=K+1$ ;  $K=8$ ;  $L=9$ ;**

**1st order:  $N=2L=18$ ;**

# Approximation Error

**Approximation of Response Surface: AA12.prj**

Experiment No. Total=90

Aprox Order:  Third,  Second,  First-linear

Local Approx

Buttons: Calculate Point, View Coefficients, Save Result, OK, Help, Edit Project File, Options, Approximate, View Graphs, Clear Table, Cancel

| Functions Yi:  | S5               | S1buckl           | S2buckl           | S3buckl           | S4buckl           | S1  |
|----------------|------------------|-------------------|-------------------|-------------------|-------------------|-----|
| Sigma Cross    | 2681712.928627   | 2246294.432634    | 2503833.442774    | 6311769.635414    | 14875647.381455   | 68  |
| Sigma Cross%   | 4.23%            | 10.80%            | 6.54%             | 14.14%            | 12.80%            |     |
| Sigma          | 1047003.236639   | 829526.115440     | 819511.589252     | 2168346.977214    | 5107312.004324    | 21  |
| Sigma%         | 1.650813         | 3.986571          | 2.140326          | 4.856202          | 4.395781          |     |
| Sigma0         | 1480686.177103   | 1173127.082797    | 1158964.404041    | 3066505.703106    | 7222829.903786    | 31  |
| Sigma0%        | 2.334602         | 5.637863          | 3.026878          | 6.867707          | 6.216572          |     |
| MeanExpValue   | 265500000.000000 | -166066666.666667 | -139518888.888889 | -104783333.333333 | -275088888.888889 | -21 |
| StDev of Exp   | 63423488.463572  | 20808011.277711   | 38289107.515336   | 44651084.968776   | 116186693.103475  | 76  |
| Exp. Range     | 221000000.000000 | 72000000.000000   | 134900000.000000  | 161500000.000000  | 360000000.000000  | 27  |
| MaxError       | 2408850.640445   | 1735052.052047    | -1846892.758088   | 4931848.640895    | 11888320.749038   | -54 |
| Bad Point No.  | 56               | 55                | 12                | 64                | 14                | 6   |
| Max Rel Error  | 1.00%            | 1.01%             | 1.79%             | 9.14%             | 7.45%             |     |
| BadRelPointNo. | 69               | 55                | 37                | 64                | 64                | 3   |

Filter Experiment for All Functions:  1,  2,  3,  4,  5,  6,  7,  8,  9,  10,  11,  12,  13,  14,  15,  16,  17,  18,  19,  20,  21,  22,  23,  39,  40,  41,  42,  43,  44,  45,  46,  47,  48,  49,  50,  51,  52,  53,  54,  55,  56,  57,  58,  59,  60,  61,  77,  78,  79,  80,  81,  82,  83,  84,  85,  86,  87,  88,  89,  90

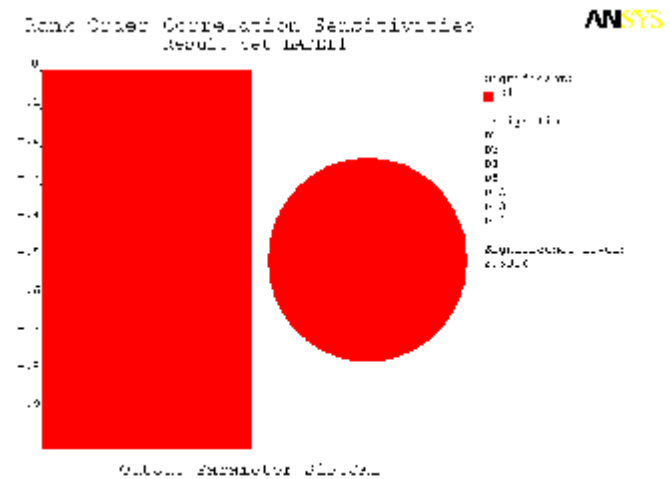
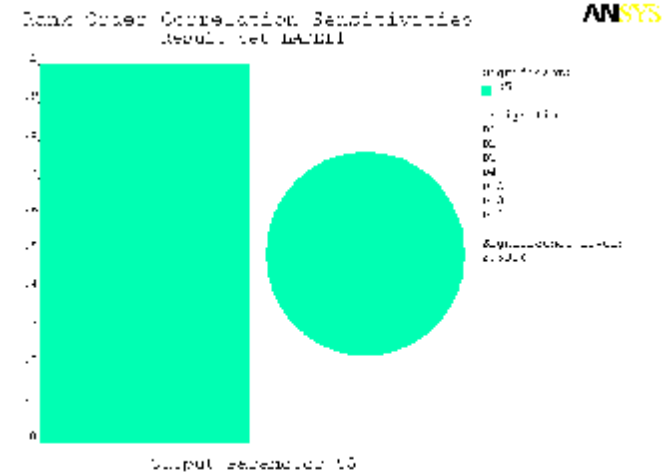
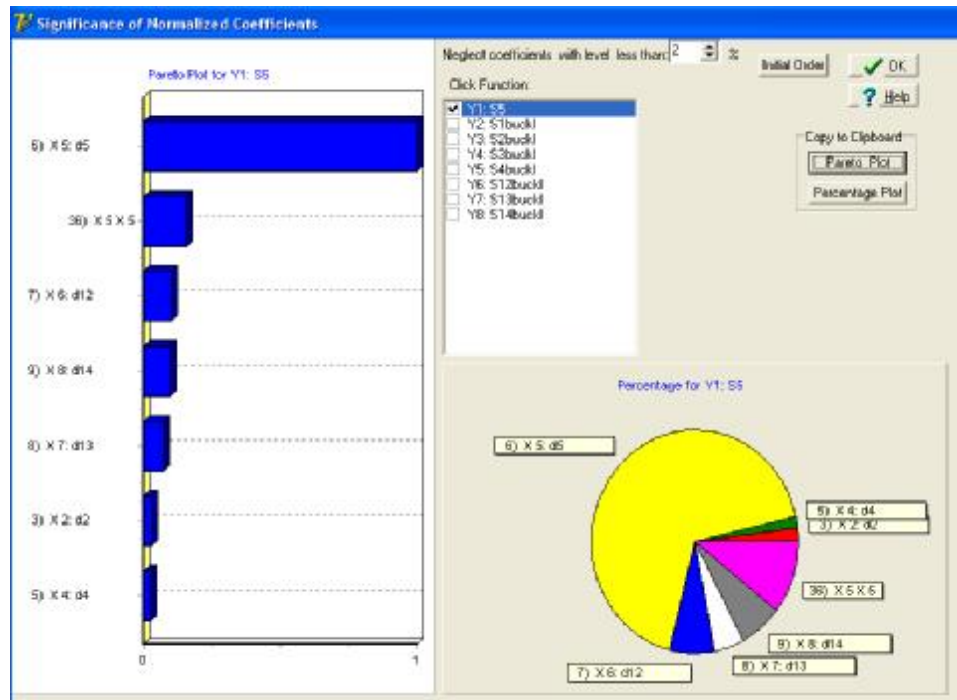
Input No to Filter Experiment for Each Function Separately: Residuals, Automatic Elimination, View Elimination Graphs

## 2<sup>nd</sup> order polynomial approximation

$$\hat{\Phi}(x) = b_0 + \sum_{i=1}^K b_i x_i + \sum_{i=1}^K b_{ii} x_i^2 + \sum_{i < j} \sum_{j=2}^K b_{ij} x_i x_j + e$$

## Cross-Validation % error

# Meta-model Sensitivity Analysis



# Optimum Solution with RSM

Optimization: AA12.prj

Criterion:  $+4.243 \times X6 \times X6 + 4.243 \times X7 \times X7 + 4.243 \times X8 \times X8 + 0.07657632 \times 7500 = 731.778978879646$  Optimize (1) Iterations 1000000

| Indices No       | Min      | Type   | Ma  | Criterion= | 731.77898   |  |  |  |  |  | 731.77898   |
|------------------|----------|--------|-----|------------|-------------|--|--|--|--|--|-------------|
| 1) X1: d1        | 0.11     | 0.0001 | 0.1 | 1) X1=     | 0.1211      |  |  |  |  |  | 0.1211      |
| 2) X2: d2        | 0.08     | 0.0001 | 0.1 | 2) X2=     | 0.0992      |  |  |  |  |  | 0.0992      |
| 3) X3: d3        | 0.06     | 0.0001 | 0.0 | 3) X3=     | 0.0832      |  |  |  |  |  | 0.0832      |
| 4) X4: d4        | 0.07     | 0.0001 | 0.0 | 4) X4=     | 0.07        |  |  |  |  |  | 0.07        |
| 5) X5: d5        | 0.008    | 1E-005 | 0.0 | 5) X5=     | 0.01128     |  |  |  |  |  | 0.01128     |
| 6) X6: d12       | 0.09     | 0.0001 | 0.1 | 6) X6=     | 0.09        |  |  |  |  |  | 0.09        |
| 7) X7: d13       | 0.11     | 0.0001 | 0.1 | 7) X7=     | 0.1234      |  |  |  |  |  | 0.1234      |
| 8) X8: d14       | 0.14     | 0.0001 | 0.1 | 8) X8=     | 0.1577      |  |  |  |  |  | 0.1577      |
| 9) Y1: S5        | 0        | 3      | 2E  | 9) Y1=     | 1.9995205E  |  |  |  |  |  | 1.9995205E  |
| 10) Y2: S1buckl  | -1.6E008 | 3      | 0   | 10) Y2=    | -1.59696E00 |  |  |  |  |  | -1.59696E00 |
| 11) Y3: S2buckl  | -1.6E008 | 3      | 0   | 11) Y3=    | -1.14366E00 |  |  |  |  |  | -1.14366E00 |
| 12) Y4: S3buckl  | -1.6E008 | 3      | 0   | 12) Y4=    | -63719246   |  |  |  |  |  | -63719246   |
| 13) Y5: S4buckl  | -1.6E008 | 3      | 0   | 13) Y5=    | -1.5934605E |  |  |  |  |  | -1.5934605E |
| 14) Y6: S12buckl | -1.6E008 | 3      | 0   | 14) Y6=    | -1.5981183E |  |  |  |  |  | -1.5981183E |
|                  |          |        |     | 15) Y7=    | -1.5996772E |  |  |  |  |  | -1.5996772E |

0 Functional Constraints:

Off  
 On

1.0 %

# Optimum Solution

|                         |        |
|-------------------------|--------|
| Literature (P.Pedersen) | 707 kg |
|-------------------------|--------|

1) Direct optimisation using Excel optimisation solver

|                       |     |    |
|-----------------------|-----|----|
| <b>Weight (Excel)</b> | 714 | kg |
|-----------------------|-----|----|

2) Indirect optimisation using Response Surface Method and Design of Experiments

|                     |     |    |
|---------------------|-----|----|
| <b>Weight (RSM)</b> | 731 | kg |
|---------------------|-----|----|