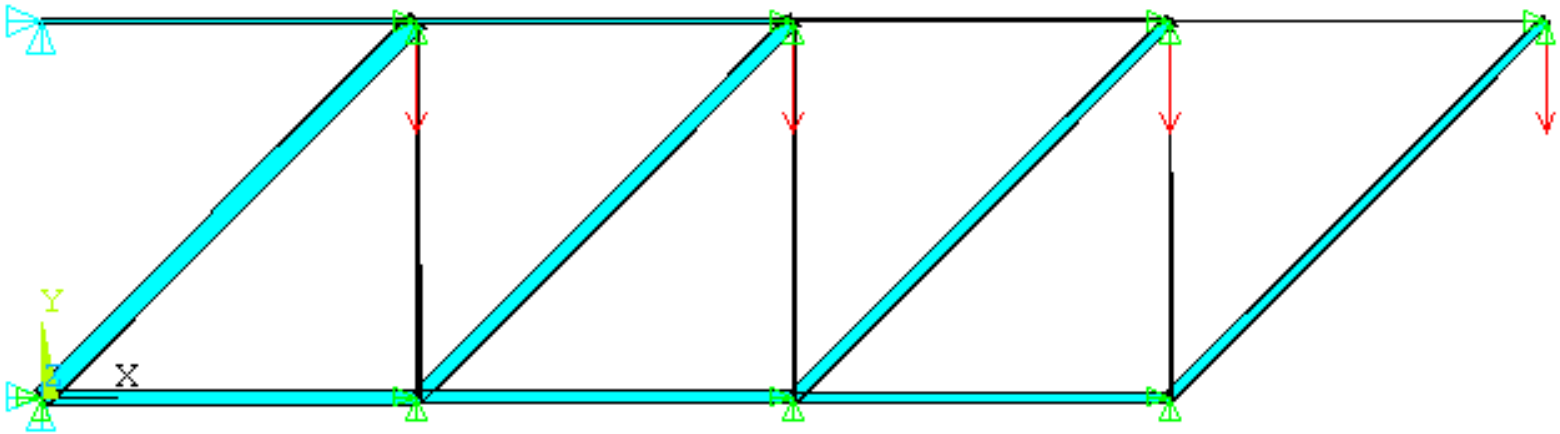
A case study

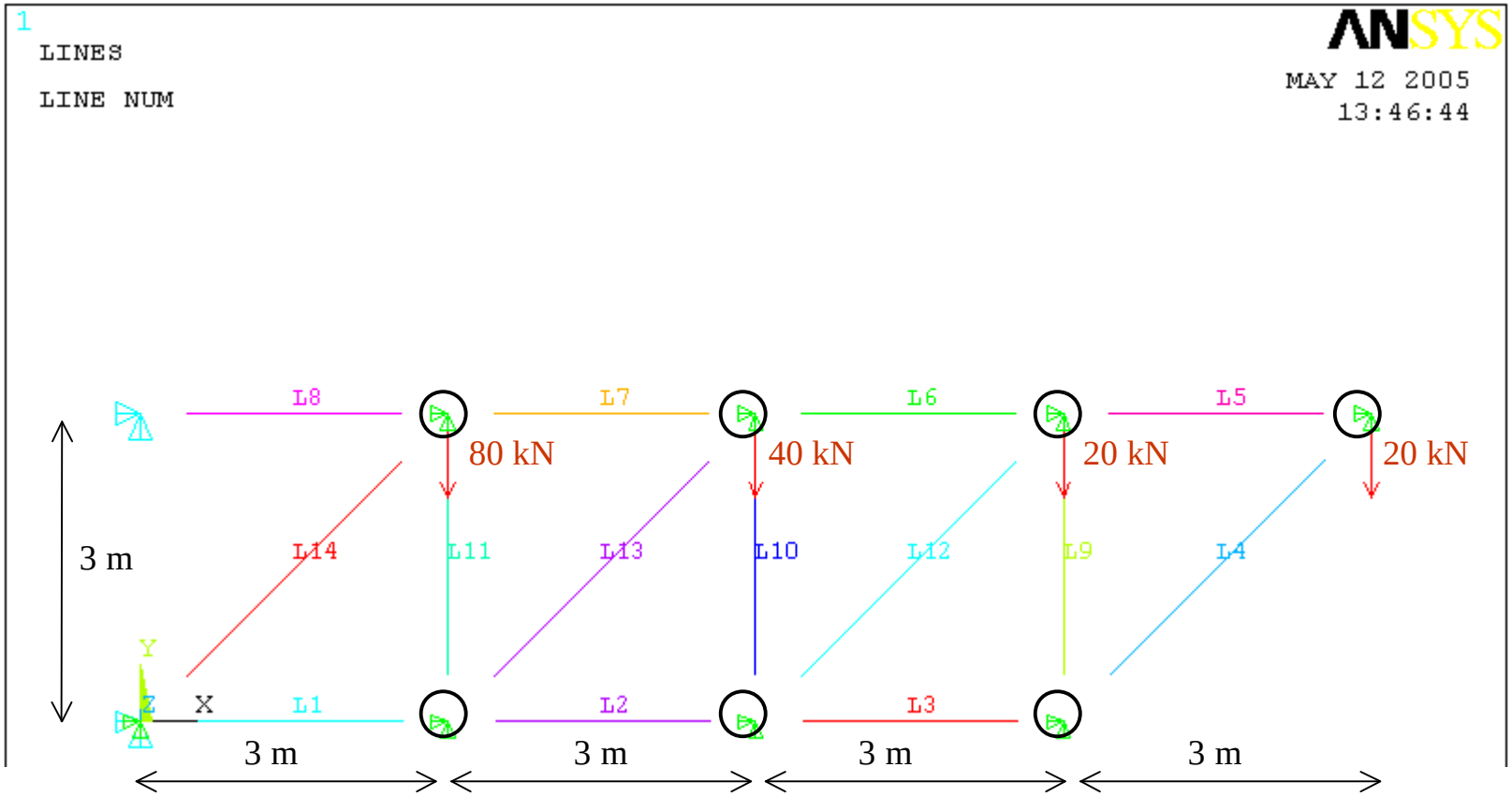
The case study example is taken from the book of

Optimal Designs - Structures and Materials - Problems and Tools -

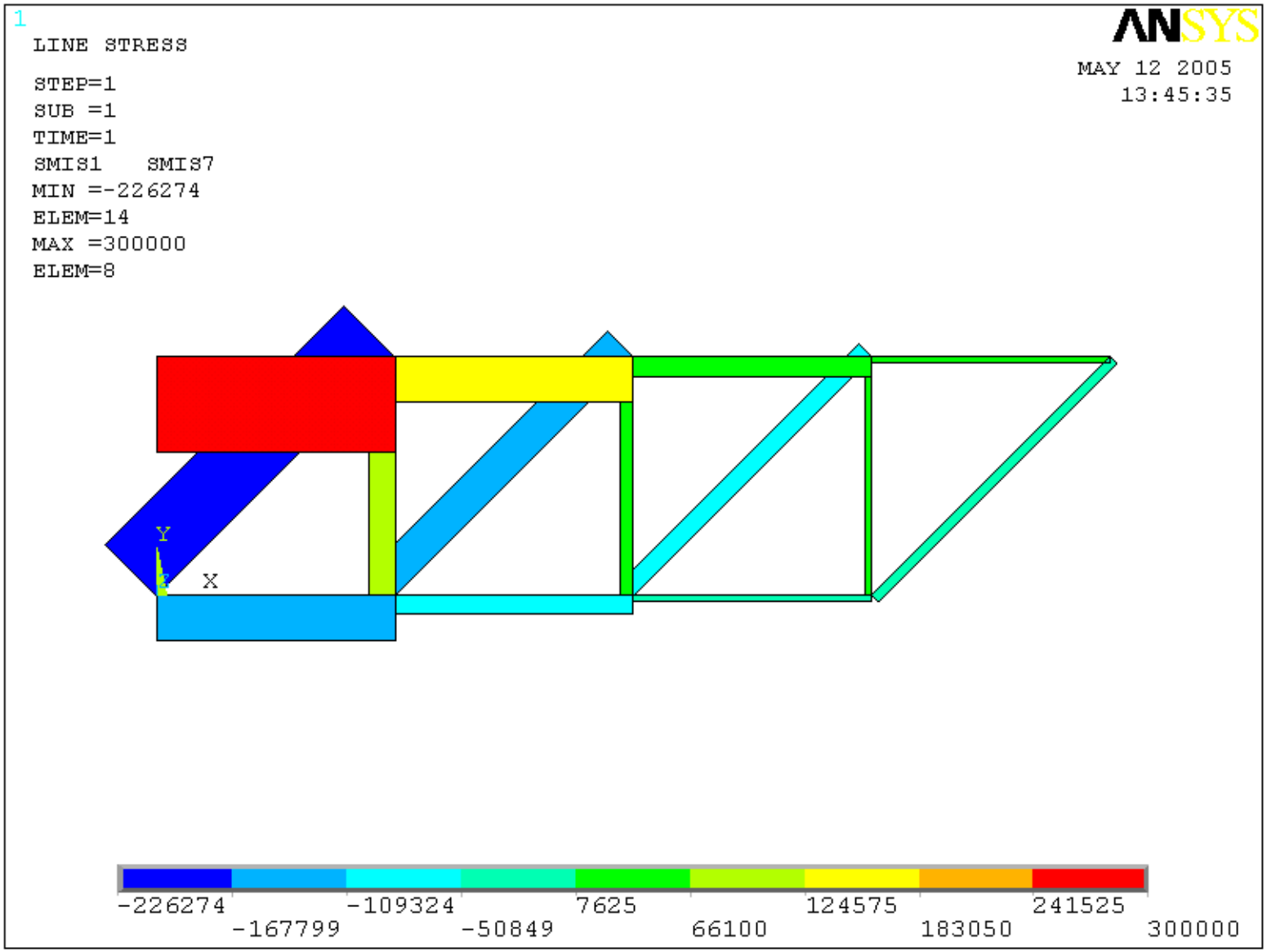
Pauli Pedersen (2003), 33-35 p.



Numbering of truss elements

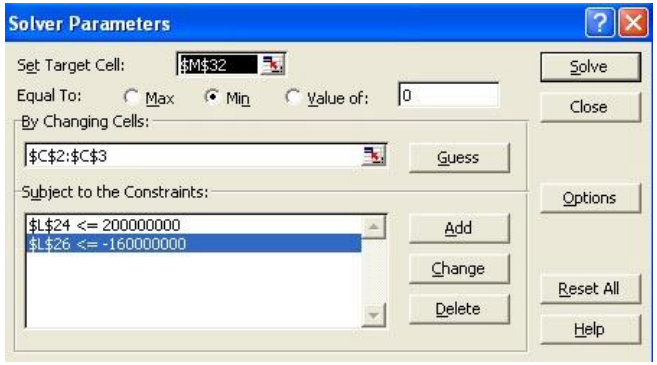


An axial force distribution in truss strucutre

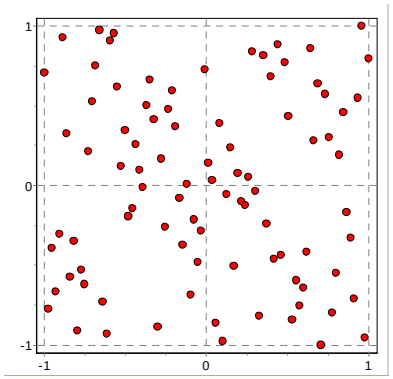


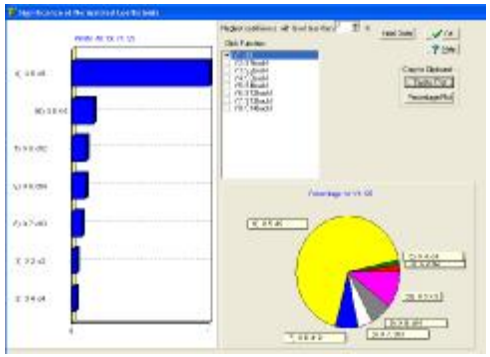
Optimisation strategy

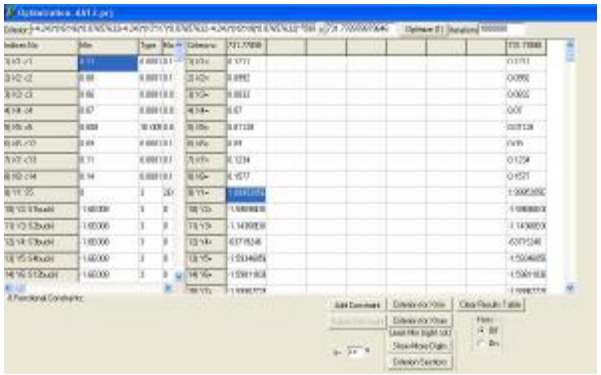
1) Direct optimisation using Excel optimisation solver



2) Indirect optimisation using Response Surface Method and Design of Experiments







Estimation of buckling criteria by reduction coefficient as function of columns slenderness

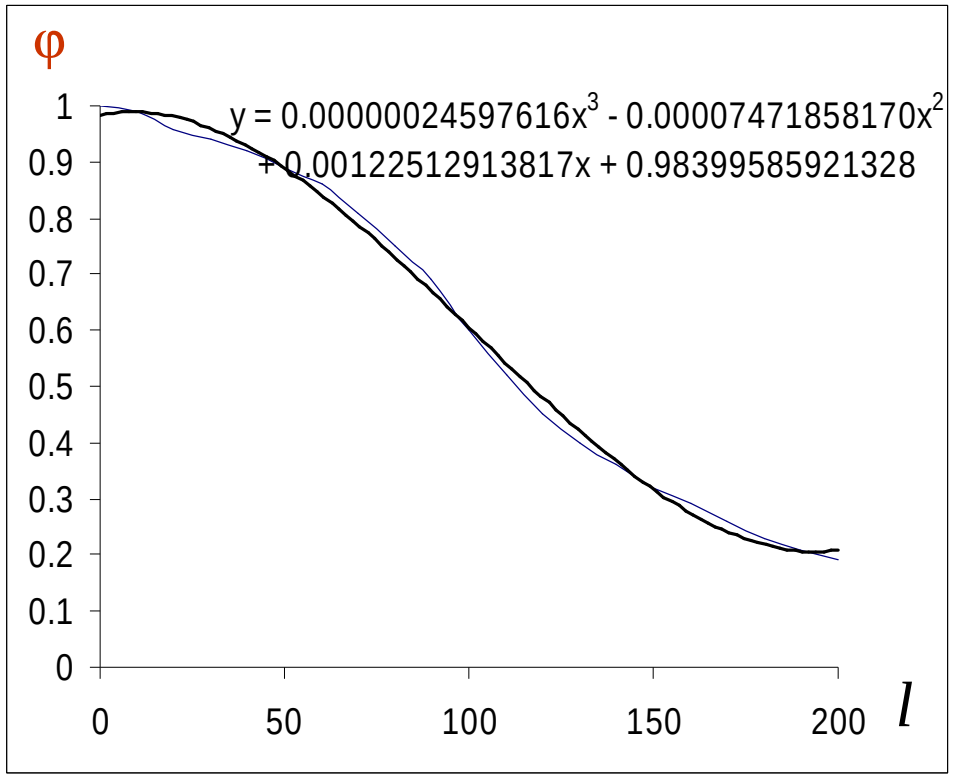
$$s \geq \frac{P}{A}$$ Strength criteria

$$s \geq \frac{P}{\varphi A}$$ Buckling criteria

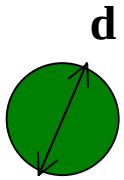
$$l_{red} = l \times m$$

$$l = \frac{l_{red}}{i}$$

$$i = \sqrt{\frac{I}{A}}$$



Optimum cross-section estimation by EXCEL



d – diameter of the solid section bar

A – cross-section area

I – cross-section moment of inertia

do – outer diameter of the hollow section bar

di – inner diameter of the hollow section bar

t – wall thickness of the hollow section bar

λ – slenderness of the hollow section bar

i – radius of inertia of the hollow section bar

φ – buckling reduction coefficient

Element number	d m	A m^2	I m^4
d5;d9	0.011	0.00010	7.96E-10
d6	0.020	0.00030	7.16E-09
d7	0.030	0.00070	3.90E-08
d8	0.044	0.00150	1.79E-07
d10	0.016	0.00020	3.18E-09
d11	0.023	0.00040	1.27E-08

Element number	d0 m	A m^2	I m^4	di m	m %	t mm
d3;d4	0.083	0.00053	4.41E-07	0.079	0.95	2.1
d2;d12	0.102	0.00079	9.73E-07	0.097	0.95	2.5
d1	0.121	0.00112	1.96E-06	0.115	0.95	3.0
d13	0.123	0.00115	2.06E-06	0.117	0.95	3.1
d14	0.158	0.00191	5.64E-06	0.150	0.95	3.9

Element number	λ	i mm	φ
s3;s4	147.5	2.88	0.33
s2;s12	121.0	3.51	0.47
s1	71.8	4.18	0.78
s13	100.3	4.23	0.60
s14	78.0	5.44	0.74

Optimum design using Excel solver

Design constraints – tensile stress 200MPa; compression stress –160 MPa

Element number	N kN	S Pa	S REAL Pa
T1	20000	2.00E+08	
T8	300000	2.00E+08	
C3;C4	-28000	-1.60E+08	-5.25E+07
C2;C12	-60000	-1.60E+08	-7.58E+07
C1	-140000	-1.60E+08	-1.24E+08
C13	-111300	-1.60E+08	-9.66E+07
C14	-226300	-1.60E+08	-1.19E+08

N - Axial force in bar element

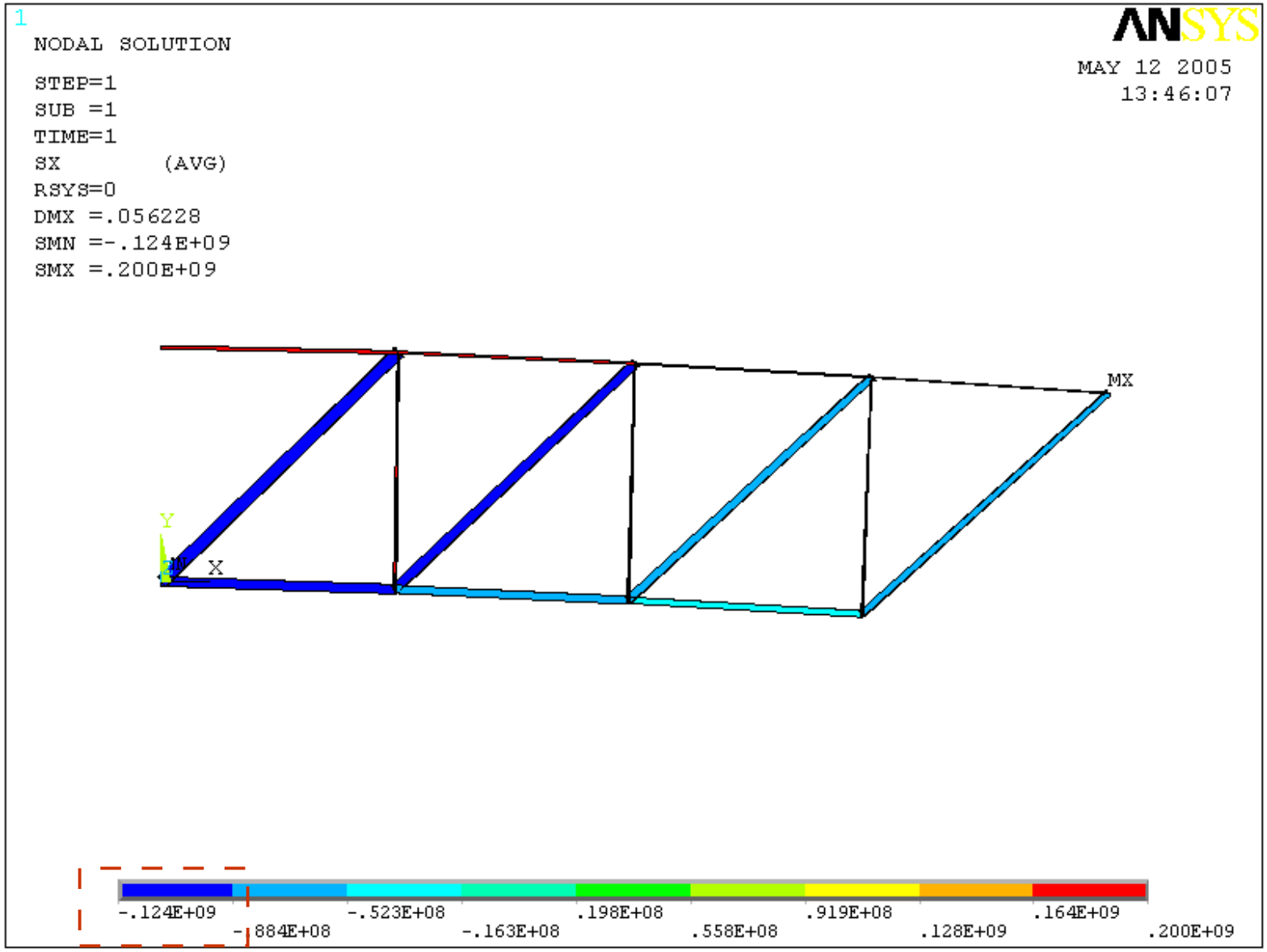
σ– Stress in bar element with buckling reduction coefficient

σREAL –Real stress in bar without buckling reduction coefficient

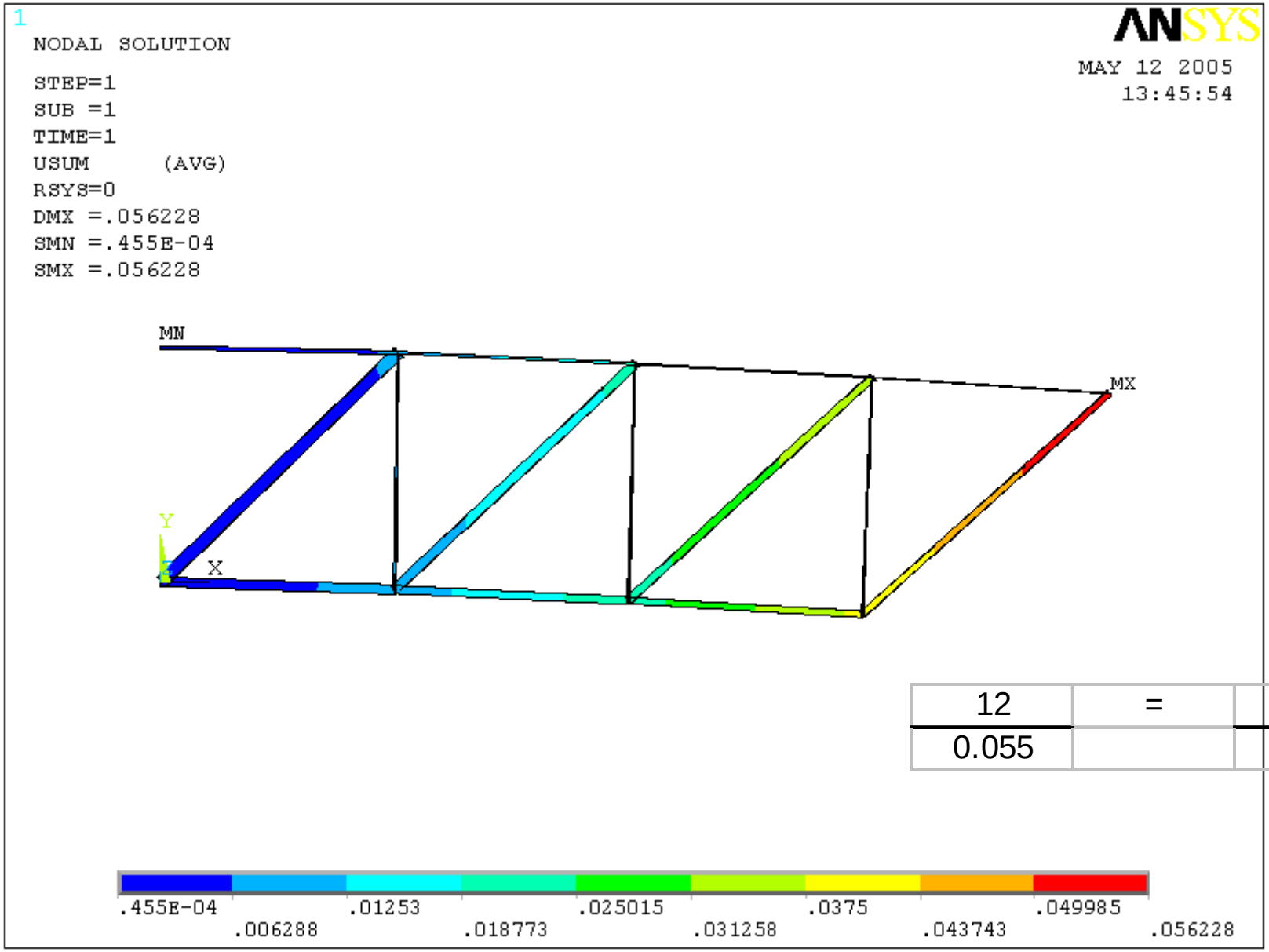
Cost function – minimisation of truss weight

Weight	714.4	kg
Volume	0.095	m^3

Verification with FEM ANSYS normal stresses distribution



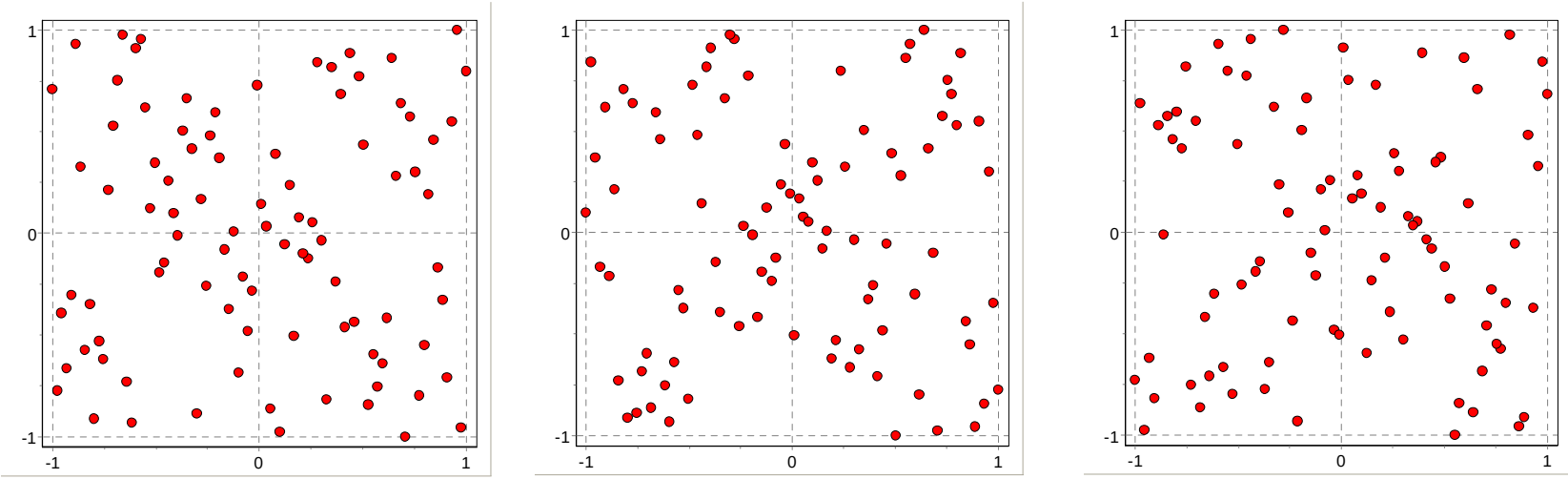
Verification with FEM ANSYS deflection



12	=	1
0.055		218

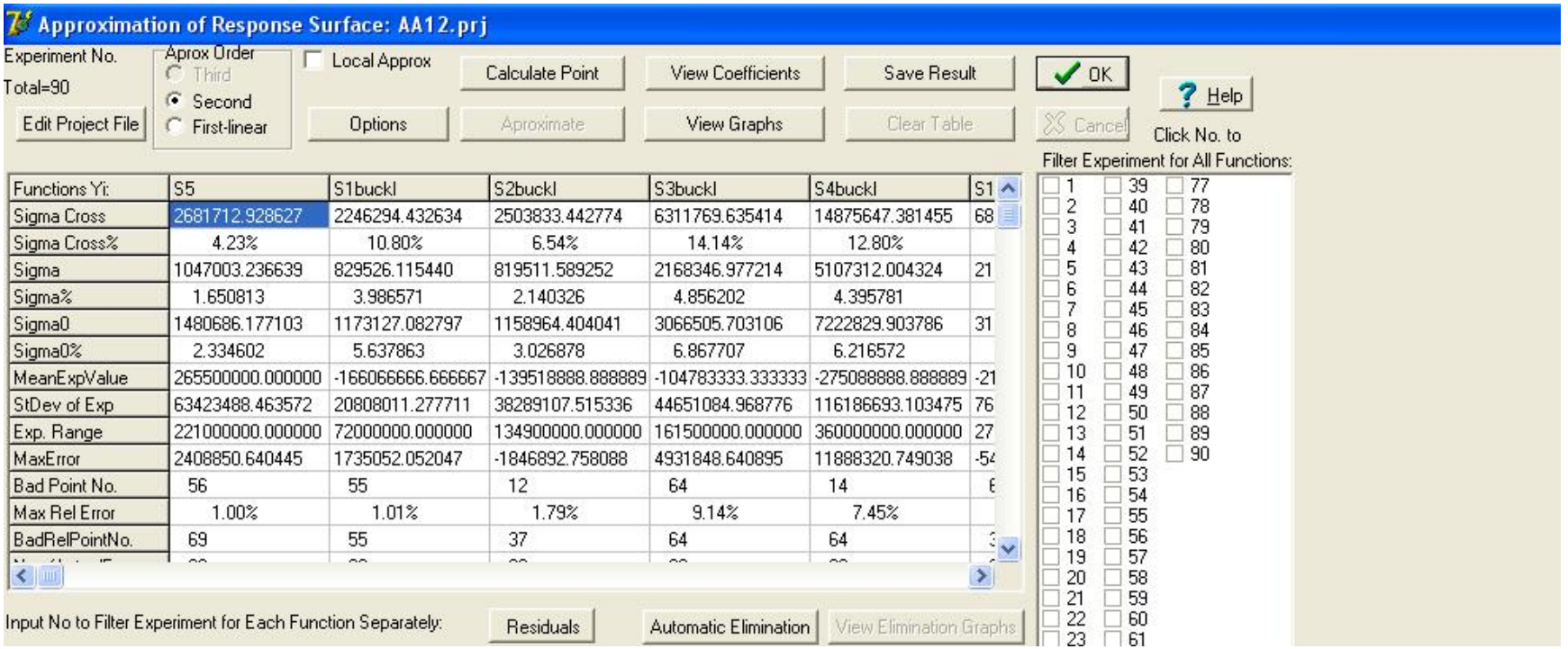
Design of Experiments

Latin – Hypercube - plan of experiments



K – number of design variables
2nd order: L=1+2K+K(K-1)/2; K=8; L=45;
Sample points of ED - 2nd order: N~2L; N=90;
1st order: L=K+1; K=8; L=9;
1st order: N=2L=18;

Approximation Error



Functions Yi:	S5	S1buckl	S2buckl	S3buckl	S4buckl	S1
Sigma Cross	2681712.928627	2246294.432634	2503833.442774	6311769.635414	14875647.381455	68
Sigma Cross%	4.23%	10.80%	6.54%	14.14%	12.80%	
Sigma	1047003.236639	829526.115440	819511.589252	2168346.977214	5107312.004324	21
Sigma%	1.650813	3.986571	2.140326	4.856202	4.395781	
Sigma0	1480686.177103	1173127.082797	1158964.404041	3066505.703106	7222829.903786	31
Sigma0%	2.334602	5.637863	3.026878	6.867707	6.216572	
MeanExpValue	265500000.000000	-166066666.666667	-139518888.888889	-104783333.333333	-275088888.888889	-21
StDev of Exp	63423488.463572	20808011.277711	38289107.515336	44651084.968776	116186693.103475	76
Exp. Range	221000000.000000	72000000.000000	134900000.000000	161500000.000000	360000000.000000	27
MaxError	2408850.640445	1735052.052047	-1846892.758088	4931848.640895	11888320.749038	-5
Bad Point No.	56	55	12	64	14	6
Max Rel Error	1.00%	1.01%	1.79%	9.14%	7.45%	
BadRelPointNo.	69	55	37	64	64	

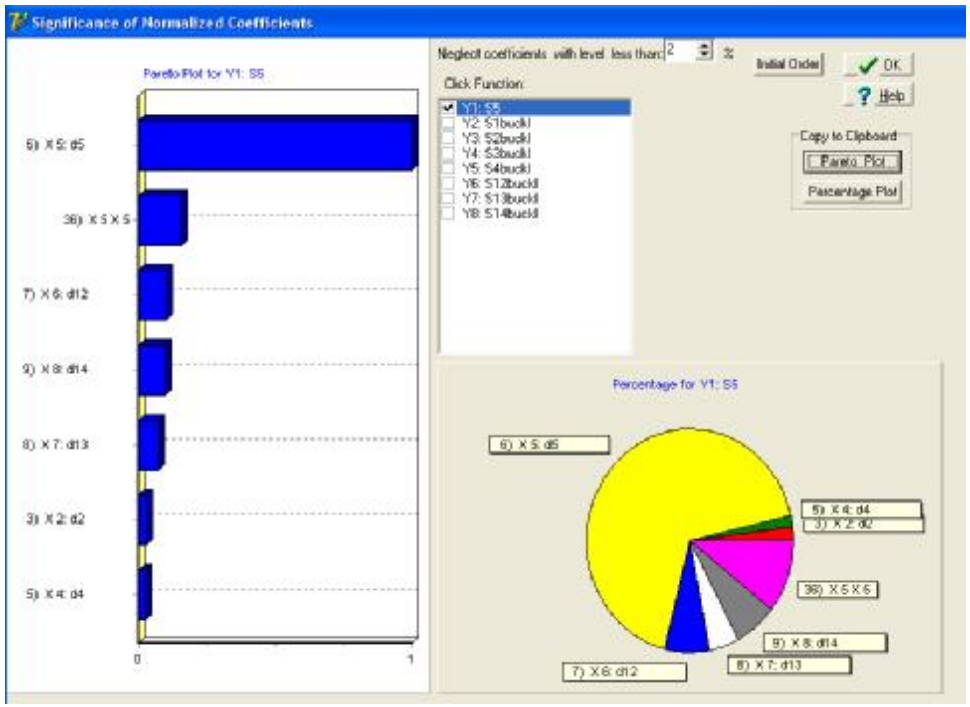
1	39	77	2	40	78	3	41	79	4	42	80
5	43	81	6	44	82	7	45	83	8	46	84
9	47	85	10	48	86	11	49	87	12	50	88
13	51	89	14	52	90	15	53		16	54	
17	55		18	56		19	57		20	58	
21	59		22	60		23	61				

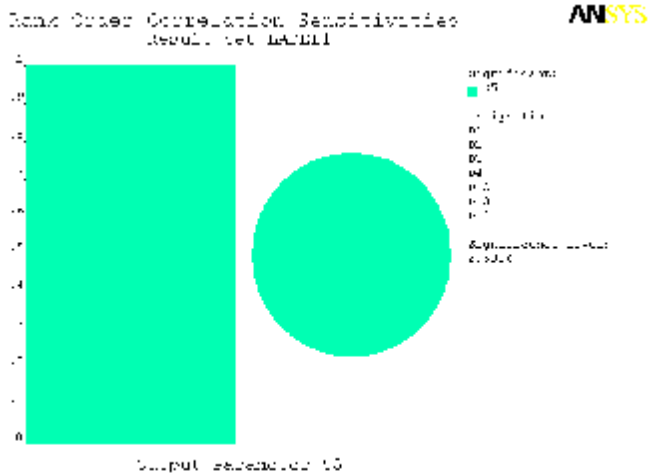
2nd order polynomial approximation

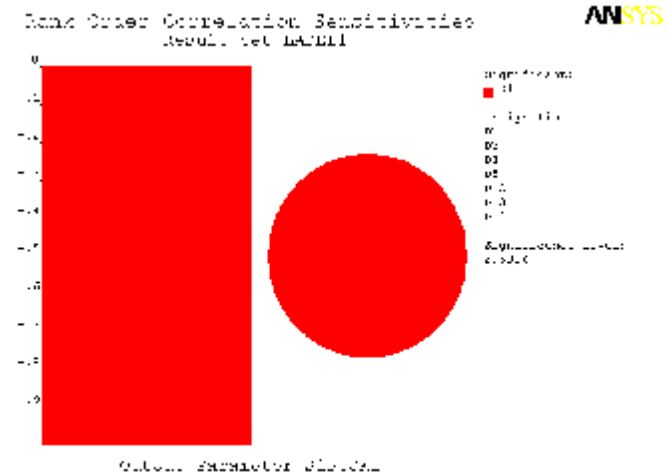
$$\hat{\Phi}(x) = b_0 + \sum_{i=1}^{K} b_i x_i + \sum_{i=1}^{K} b_{ii} x_i^2 + \sum_{i<j} \sum_{j=2}^{K} b_{ij} x_i x_j + e$$

Cross-Validation % error

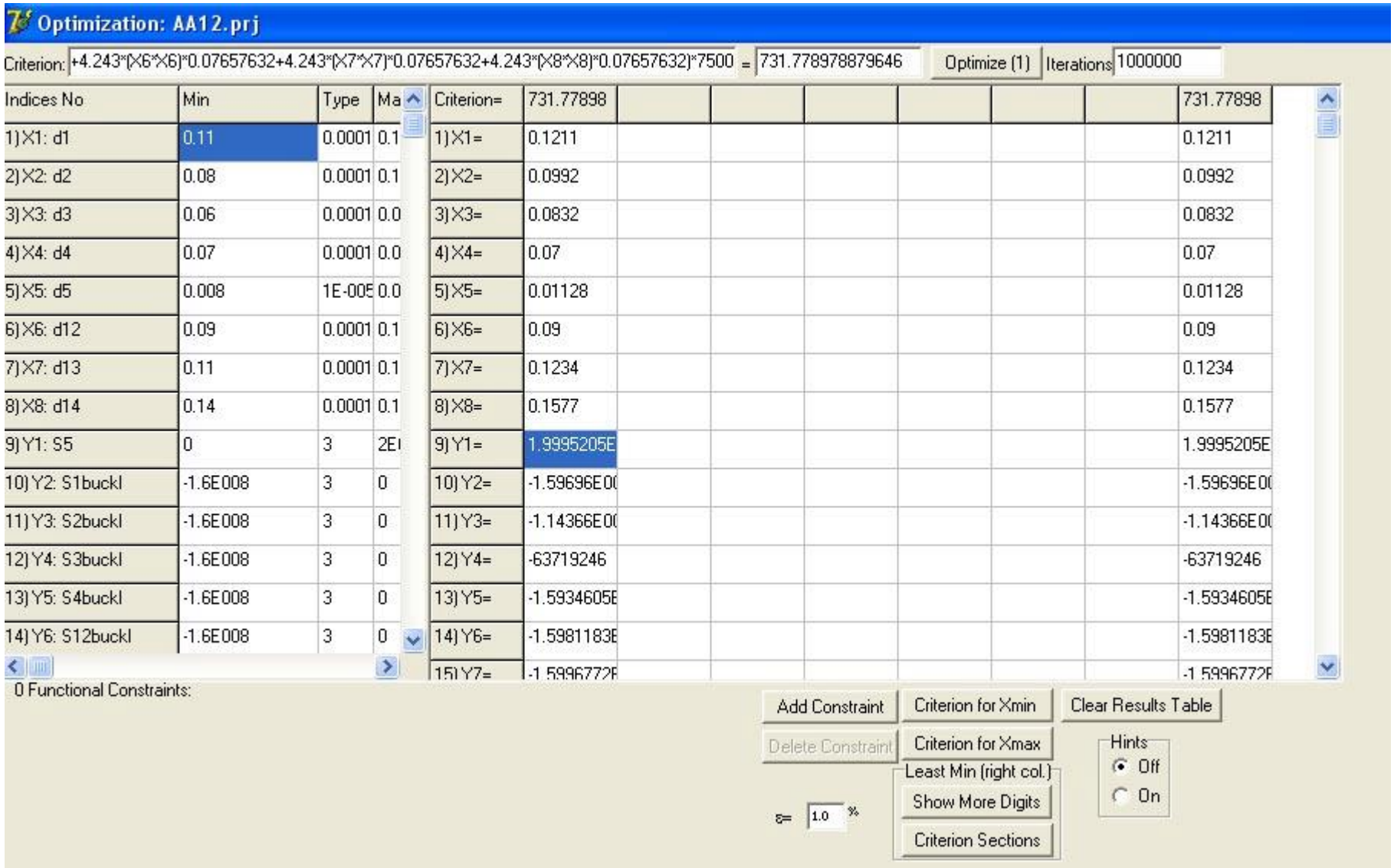
Meta-model Sensitivity Analysis







Optimum Solution with RSM



Optimum Solution

Literature (P.Pedersen)	707 kg

1) Direct optimisation using Excel optimisation solver

Weight (Excel)	714	kg

2) Indirect optimisation using Response Surface Method and Design of Experiments

Weight (RSM)	731	kg